08/12/2019
SFF 2019
1450 – 1510 Hrs.
Salon B

Part-Level Thermal Modeling in Additive Manufacturing using Graph Theory

Reza Yavari
Kevin Cole, Ph.D.
Prahalada Rao, Ph.D.

mrezayavari89@huskers.unl.edu
Mechanical and Materials Engineering
University of Nebraska-Lincoln

Acknowledgements

National Science Foundation (NSF).

CMMI 1752069

Outline

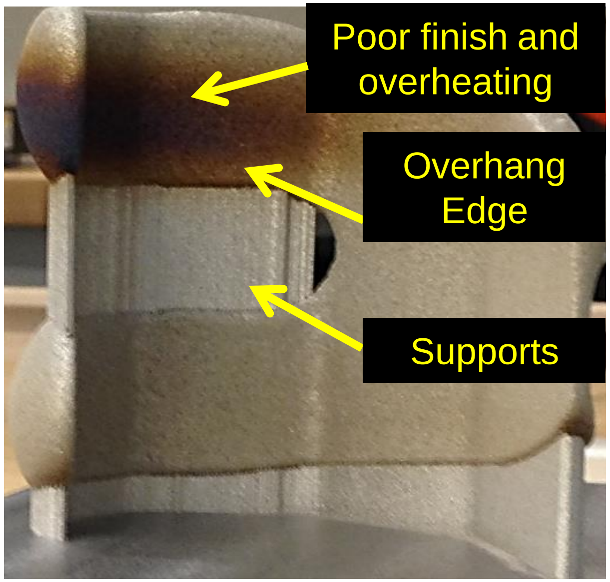
- Introduction
  - Engineering problem
  - AM thermal simulation categories
  - Solving heat equation using graph theory
- Graph theory approach in AM
- Result
  - Simulation vs exact analytical
  - Simulation vs finite element
- Conclusions and Future Work

Outline

- Introduction
  - Engineering problem
  - AM thermal simulation categories
  - Solving heat equation using graph theory
- Graph theory approach in AM
- Result
  - Simulation vs exact analytical
  - Simulation vs finite element
- Conclusions and Future Work

Research Goal

Explain and quantify the thermal phenomena that influence the quality of metal parts made using additive manufacturing processes (metal AM).

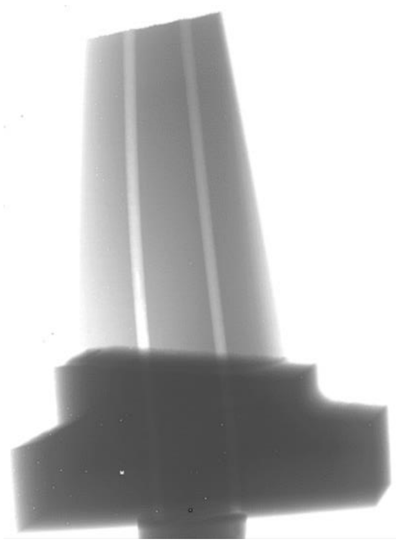


Metal AM Knee Implant

Part quality (geometry, microstructure, surface finish) in metal AM is governed by the magnitude and direction of heat flow during printing.

Engineering Problem

- It will take hours, if not days and lot of money to qualify a new part using empirical testing.
- One inch tall turbine blade takes over 3 hours to X-ray CT (XCT).



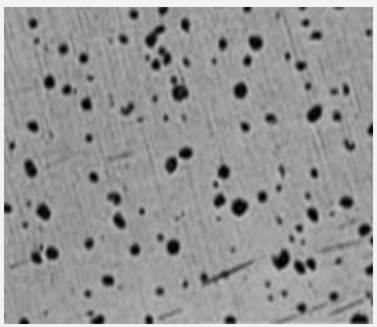
AM simulation is required to analyze the process in advance.

M. Gorelik, "Additive Manufacturing in the Context of Structural Integrity," *International Journal of Fatigue,* vol. 94, Part 2, pp. 168-177, 2017.

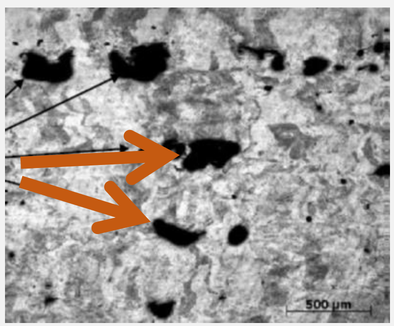
AM Thermal Simulation

1. Meltpool or small-scale modeling (< 100 µm)
   Focuses on heat source interaction zone (melt-pool)
2. Part-scale modeling (> 100 µm)

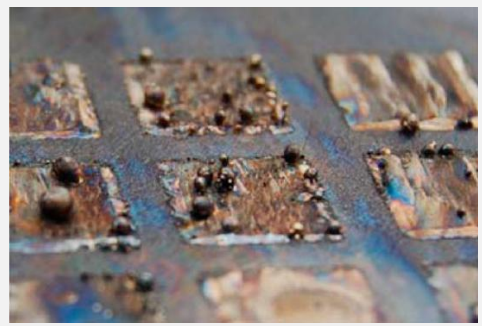
Meltpool Scale



< 10 µm
Vaporization



10 µm – 100 µm
Melting/Fusion



100 µm – 200 µm
Meltpool dynamics

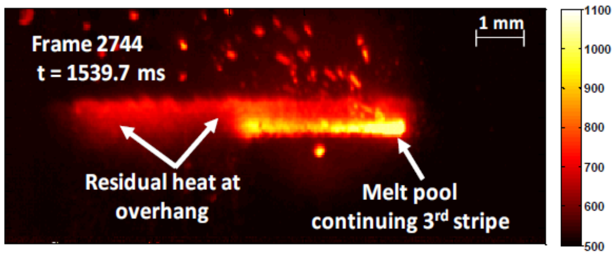
Part- Scale Modeling

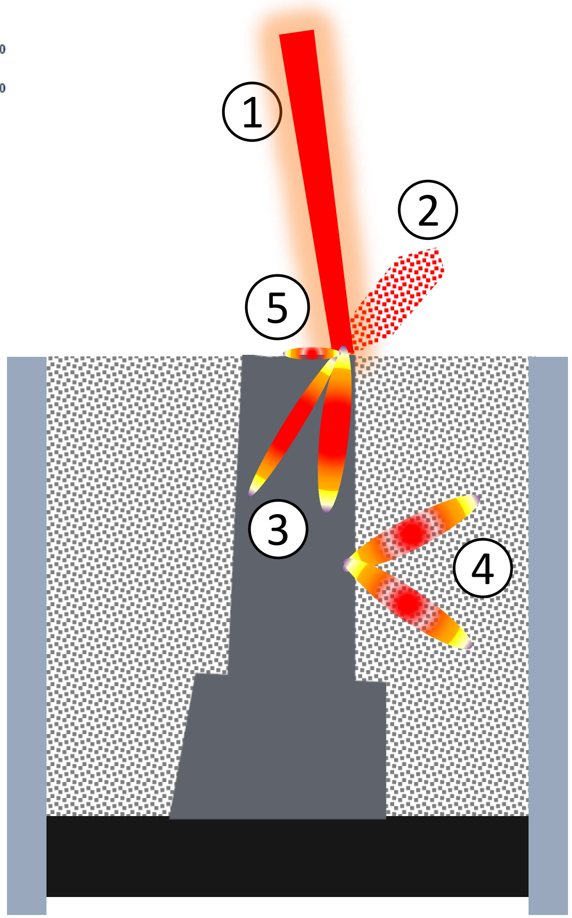
> 200 µm
Thermal-induced cracking and distortion

Focuses on predicting part-level phenomena

W. J. Sames, F. List, S. Pannala, R. R. Dehoff, and S. S. Babu, "The Metallurgy and Processing Science of Metal Additive Manufacturing," International Materials Reviews, vol. 61, pp. 315-360, 2016.

Part-Scale Modeling





1) Energy supplied by the laser to melt a unit volume of powder
2) Radiation on the top layer (part to air)
3) Conduction within the part (within part)
4) Convection between part and surrounding area
5) Latent heat at the melt-pool.
6) Temperature dependent properties

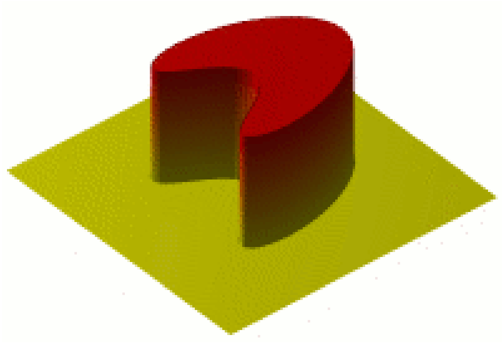
Including all the above thermal effects in a model is computationally expensive

Simplify the analysis by removing radiation and latent heat aspects.

The Scientific Problem

Temperature (T) is a function of space (x, y, z) and time (t)

$T(x, y, z, t)$



The Heat Equation (Fourier's Law of Conduction)

$$\rho c_p \frac{\partial \mathrm{T}(x,y,z,t)}{\partial t} - k\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\mathrm{T}(x,y,z,t) = 0$$

$$\rho c_p \frac{\partial \mathrm{T}}{\partial t} - k\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\mathrm{T} = 0$$

K = thermal conductivity ρ = density C_p = specific heat

Yavari, M. R., Cole, K. D., & Rao, P. (2019). Thermal Modeling in Metal Additive Manufacturing Using Graph Theory. Journal of Manufacturing Science and Engineering, 141(7), 071007.

$$\frac{\partial \mathrm{T}}{\partial t} - \frac{k}{\rho c_p}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathrm{T} = 0$$

Laplacian operator

$$\Delta \stackrel{\text{def}}{=} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

Continuum heat equation

$$\frac{\partial \mathrm{T}}{\partial t} - \alpha(\Delta)\mathrm{T} = 0$$

$k/\rho\, c_p = \alpha$ (Thermal diffusivity)

Hypothesis

The Heat Equation is solved as a function of the
Eigenvalues (Λ) and Eigenvectors (ϕ) of the Discrete Laplacian Matrix ($\mathcal{L}$)

$$\frac{\partial \mathrm{T}}{\partial t} - \alpha(\Delta)\mathrm{T} = 0$$

The continuous Laplacian operator is approximated by the Graph Laplacian.

$$\Delta \approx -\mathcal{L}$$

Describing the Laplacian matrix by its eigenspectrum:

$$\mathcal{L} = \phi \lambda^{*} \phi^{-1}$$

$$\mathrm{T} = e^{-\alpha g(\phi \Lambda \phi')t}$$

Outline of the Proof

Taylor Series Expansion

$$e^{-\alpha g(\phi\Lambda\phi')t} = I + \frac{(-\alpha g(\phi\Lambda\phi')t)}{1!} + \frac{(-\alpha g(\phi\Lambda\phi')t)^2}{2!} + \frac{(-\alpha g(\phi\Lambda\phi')t)^3}{3!} + \cdots$$

$$e^{-\alpha g(\phi\Lambda\phi')t} = I - \alpha g t\frac{\phi\Lambda\phi'}{1!} + \alpha^2 g^2 t^2 \frac{(\phi\Lambda\underbrace{\phi')(\phi}_{=1}\Lambda\phi')}{2!} - \alpha^3 g^3 t^3 \frac{(\phi\Lambda\underbrace{\phi')(\phi}_{=1}\Lambda\underbrace{\phi')(\phi}_{=1}\Lambda\phi')}{3!} + \cdots$$

Eigenvectors are Orthogonal $\phi\,\phi' = \mathrm{I}$

$$e^{-\alpha g(\phi\Lambda\phi')t} = I - \frac{\phi\Lambda\alpha g t\phi'}{1!} + \frac{\phi(\Lambda\alpha g t)^2\phi'}{2!} - \frac{\phi(\Lambda\alpha g t)^3\phi'}{3!} + \cdots = \phi e^{-\alpha g(\Lambda t)}\phi'$$

$$\mathrm{T} = \phi e^{-\alpha g(\Lambda)t}\phi'$$

Advantages

1. Freedom to discretize time t into any desired length.
2. Does not require matrix inversion; only matrix multiplication.
3. No meshing steps.

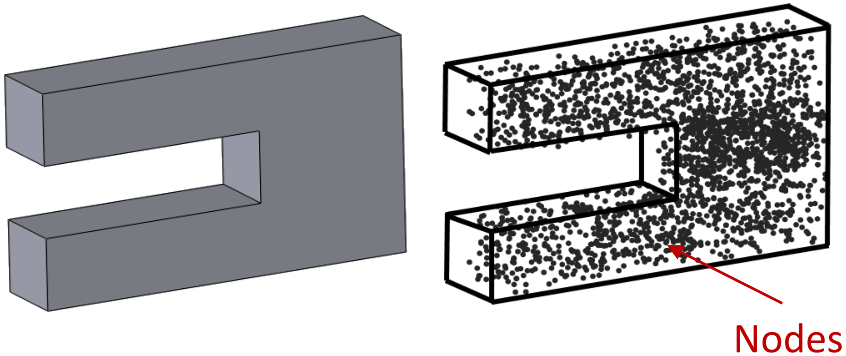
$$\mathrm{T} = \phi e^{-\alpha \mathrm{g}(\Lambda) t} \phi' \mathrm{T}_o$$

How to obtain ϕ and Λ?

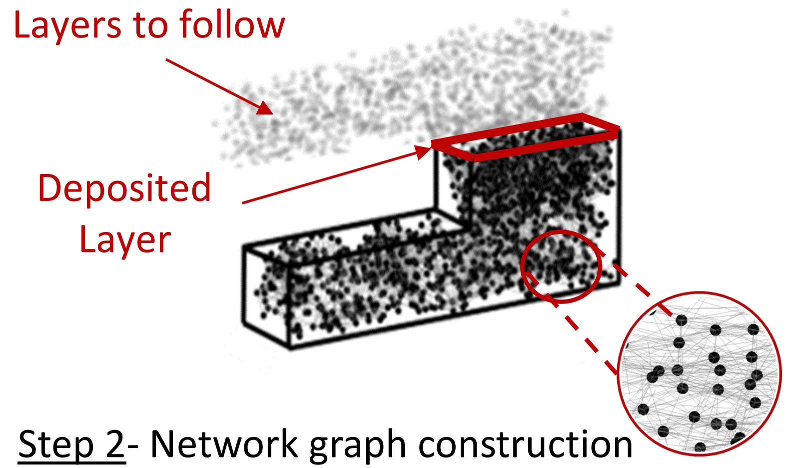
Outline

- Introduction
  - Engineering problem
  - AM thermal simulation categories
  - Solving heat equation using graph theory
- Graph theory approach in AM
- Result
  - Simulation vs exact analytical
  - Simulation vs finite element
- Conclusions and Future Work

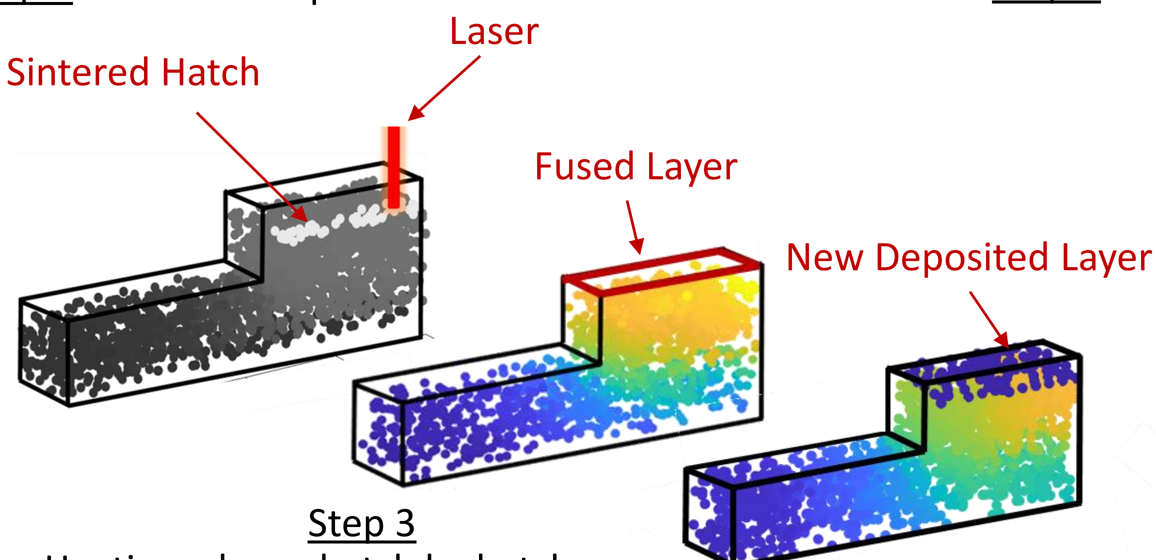
Graph Theory Approach in AM



<u>Step 1</u>- Convert the part into a set of discrete nodes

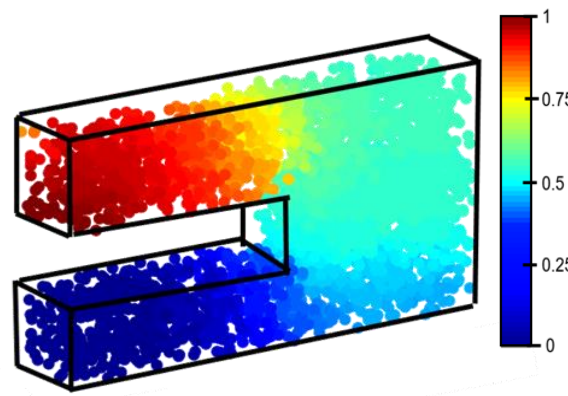


<u>Step 2</u>- Network graph construction



<u>Step 3</u>

- Heating a layer, hatch by hatch,
- Diffusion of the heat through the part
- Deposition of a new layer

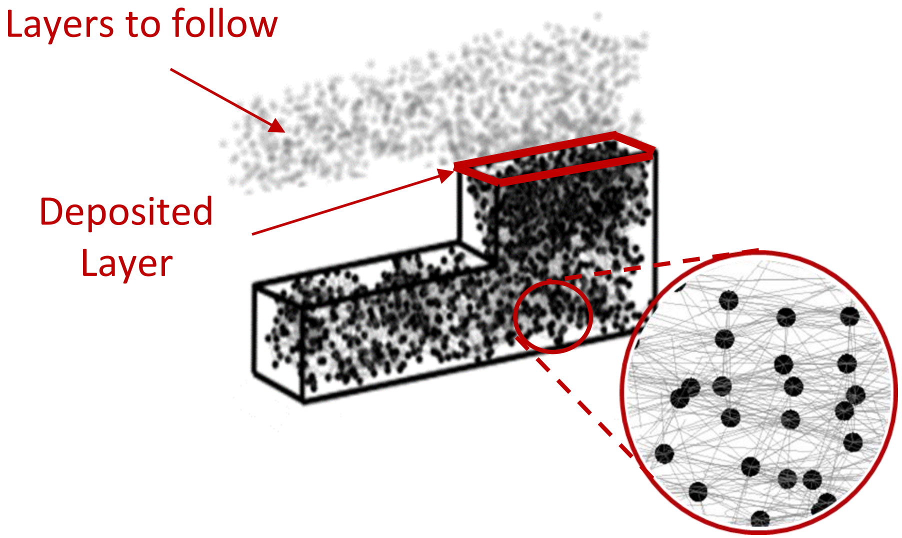


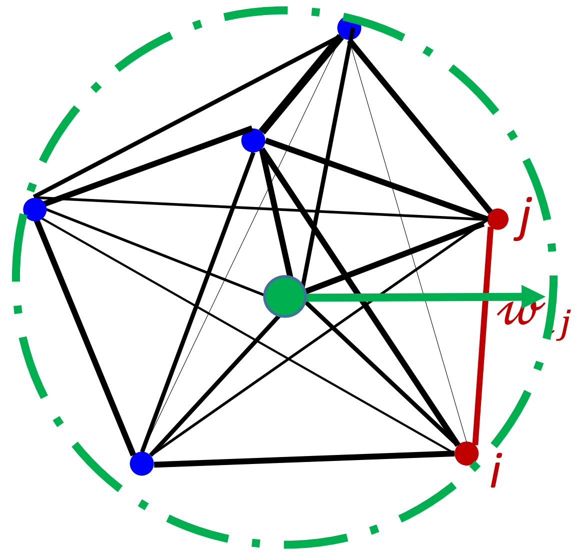
<u>Step 4</u>

Result as temperature matrix which shows the temperature history of the part

Obtaining the Laplacian Matrix ($\mathcal{L}$)

Connect nodes with a radius of ε mm





Step 2- Network graph construction

Find the Gaussian distance between nodes (Closer nodes have higher edge weights)

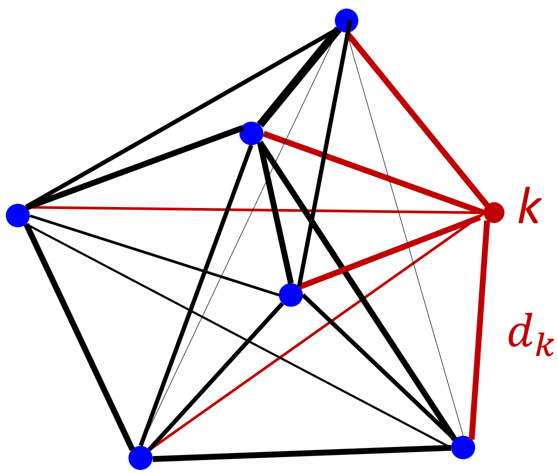
$$w_{ij} = e^{-\frac{(\vec{x}_i - \vec{x}_j)(\vec{x}_i - \vec{x}_j)^{\mathrm{T}}}{\sigma^2}}$$

Similarity matrix $S^{M \times M} \stackrel{\text{def}}{=} [w_{ij}]$

Obtaining Eigenvectors (ϕ) and Eigenvalues (Λ)

Similarity matrix

$$S^{M\times M} \stackrel{\text{def}}{=} [w_{ij}]$$



$$d_k = \sum_{j=1}^{j=M} w_{kj}$$

Degree matrix

$$\mathcal{D} \stackrel{\text{def}}{=} \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_k & 0 \\ 0 & 0 & d_M \end{bmatrix}$$

Matrix of Real positive numbers

Laplacian matrix

$$\mathcal{L} \stackrel{\text{def}}{=} (\mathcal{D} - S)$$

$$\mathcal{L}\phi = \Lambda\phi$$

Outline

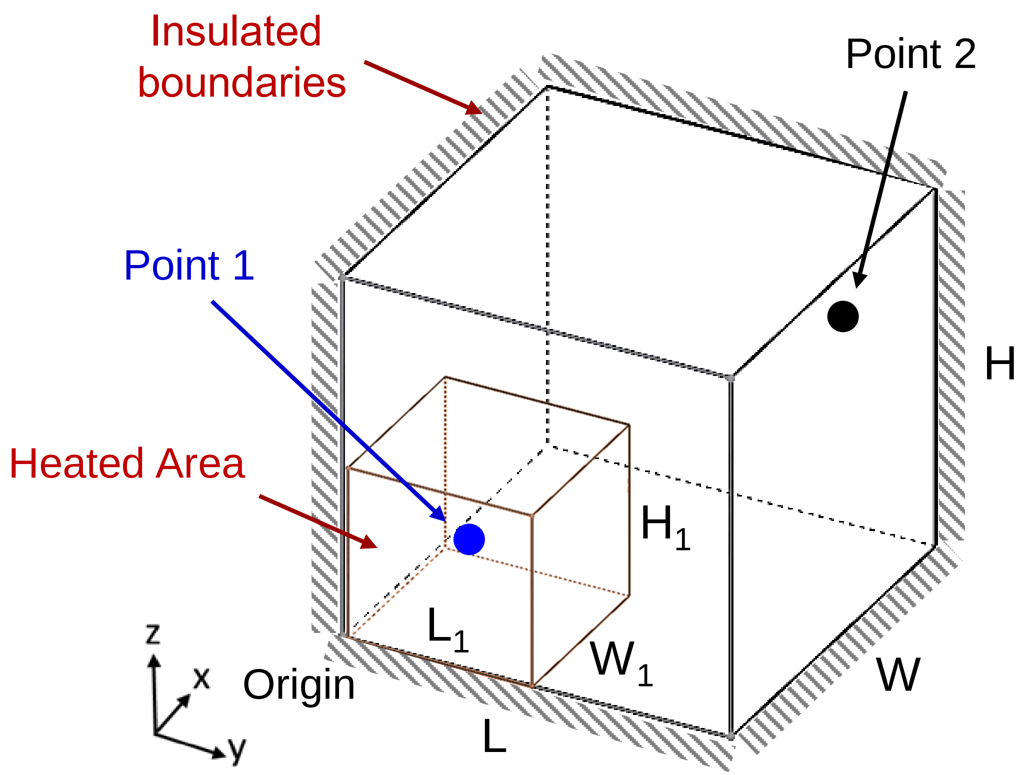
- Introduction
  - Engineering problem
  - AM thermal simulation categories
  - Solving heat equation using graph theory
- Graph theory approach in AM
- Result
  - Simulation vs exact analytical
  - Simulation vs finite element
- Conclusions and Future Work

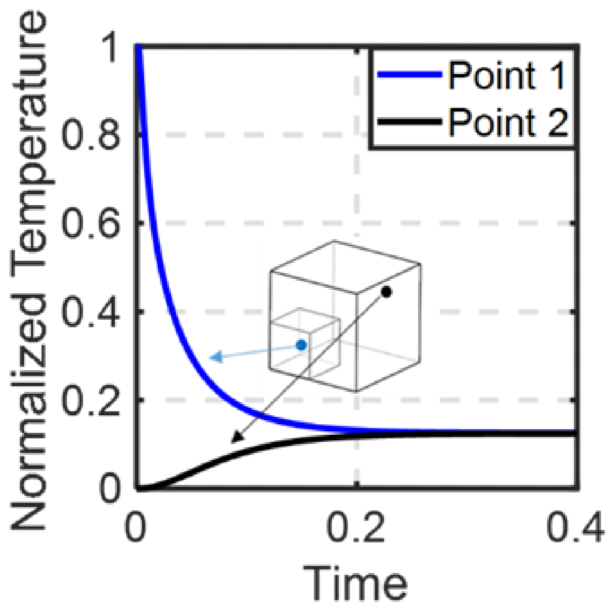
Comparison with Exact Analytical Solution

Purpose: Quantify the accuracy of graph theory diffusion with analytical solution

Geometry condition: ($W = L = H = 1$) and ($W_1 = L_1 = H_1 = 0.5$)

Observation points: Point 1 = (0.25H, 0.25L, 0.25W), Point 2 = (0.75H, 0.75L, 0.75W).

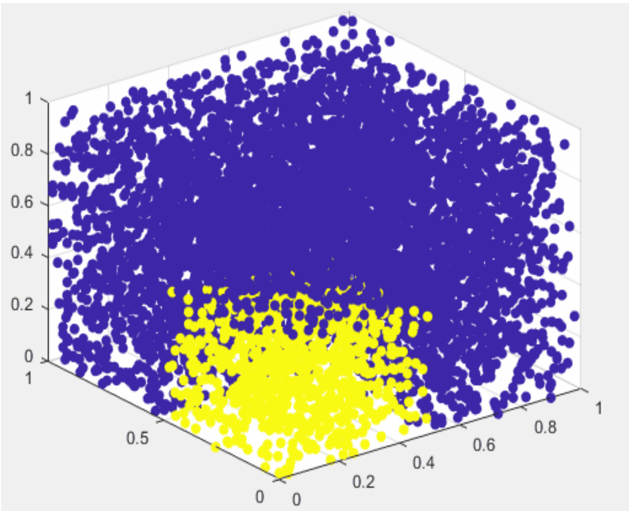


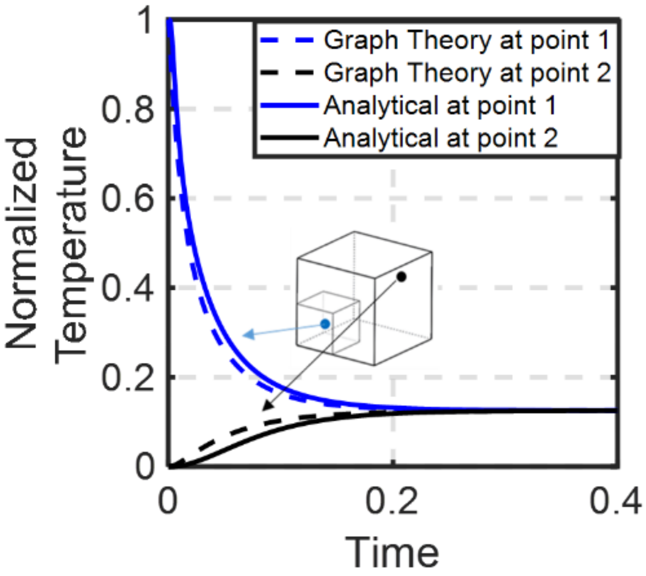


Cole et al., 2011, chap. 3. Heat Conduction Using Green's Functions, Exact Analytical Conduction Toolbox (EXACT) at UNL
www. exact.unl.edu

Comparison with Exact Analytical Solution

Graph theory captures the physics of the heat transfer for an ideal case.





Error	Graph theoretic approach time (sec.)	FE analysis time (sec.)
~ 5%	237	3,540

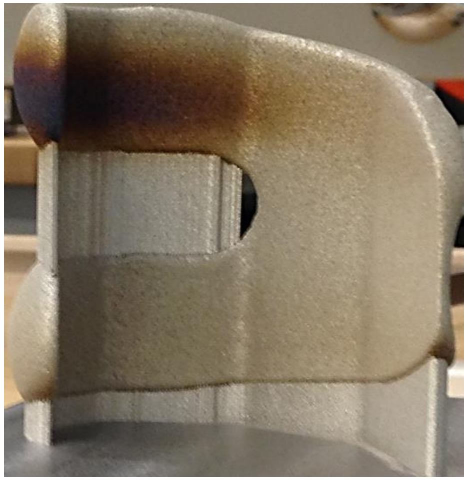
4 mins   59 mins

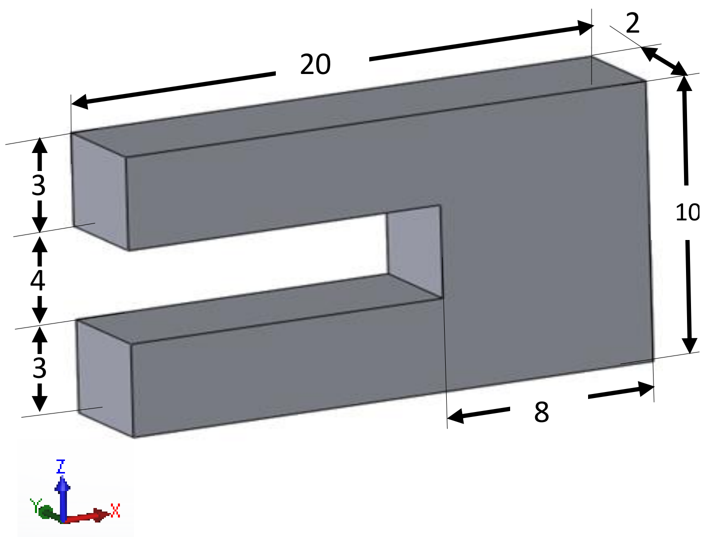
Effect of Part Geometry on Heat Flux

Understand the Causal Linkages that Govern Part Quality in Metal AM

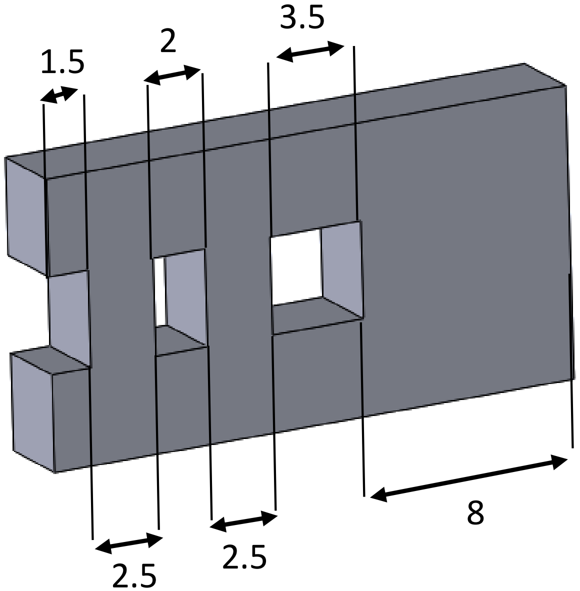
Part Geometry, Process Parameters, Material → Heat Distribution → Microstructure and Shape Flaws.

Two different part geometry studied for additive manufacturing process (LPBF).





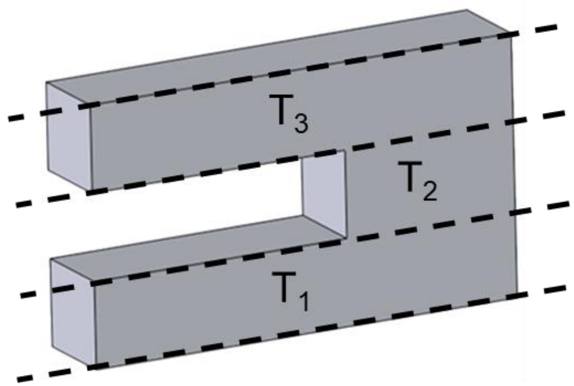
C-Shaped Part

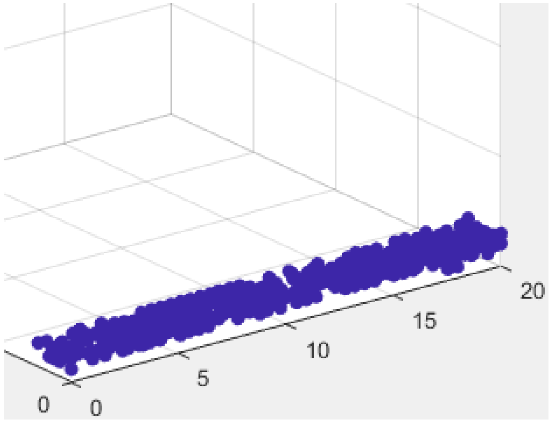


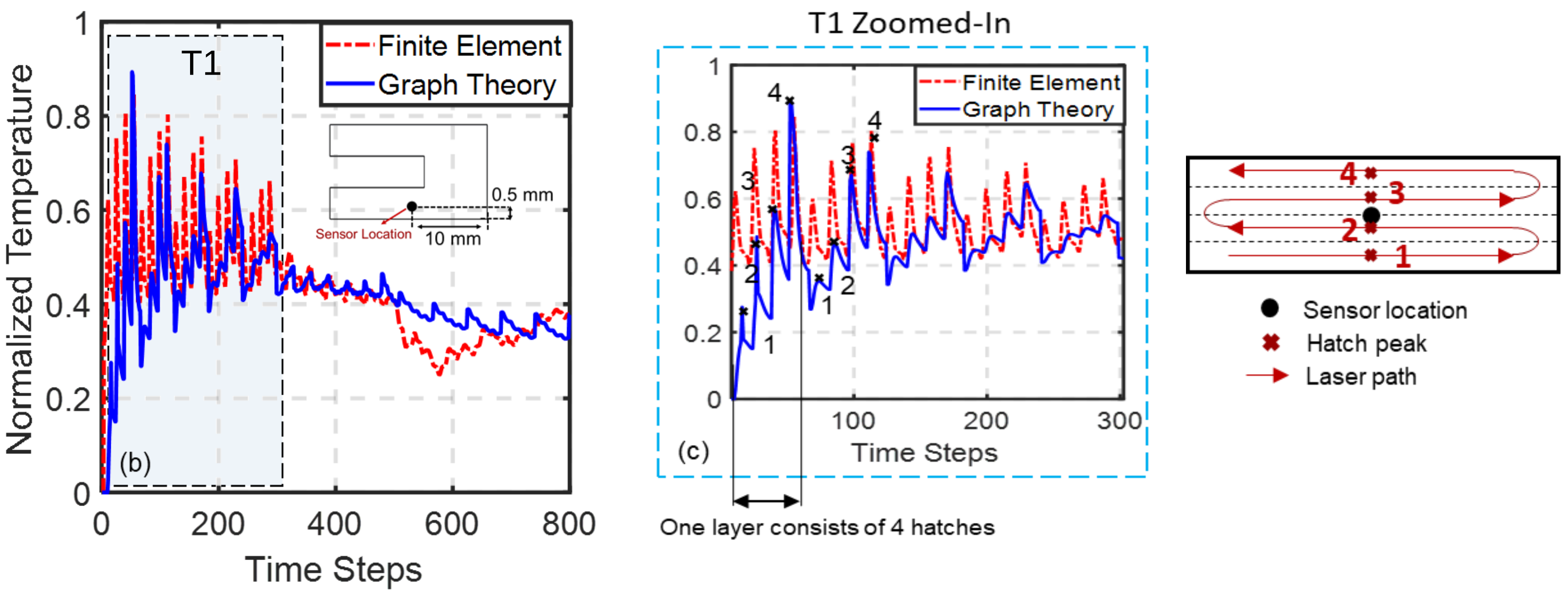
Modified C-Shaped part

Comparison with FE Analysis

Graph theory simulates the AM process in C-shaped part.



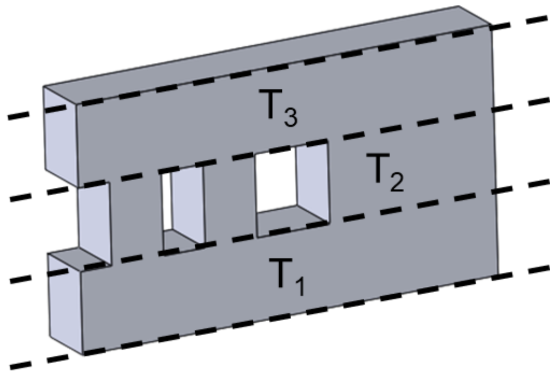


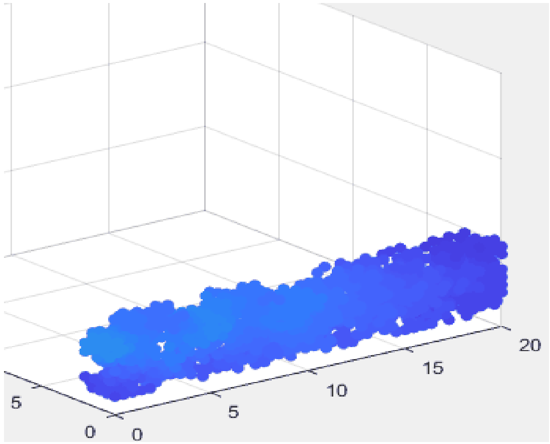


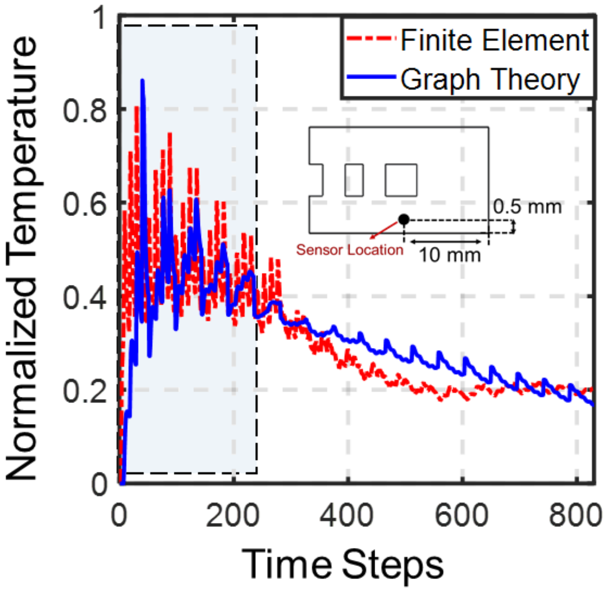
Graph theory solution converges to similar trends as finite element analysis.

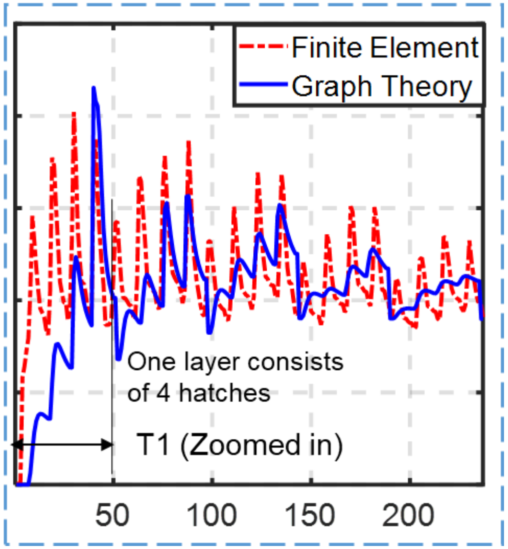
Comparison with FE Analysis

Graph theory simulates the AM process in modified C-shaped part.



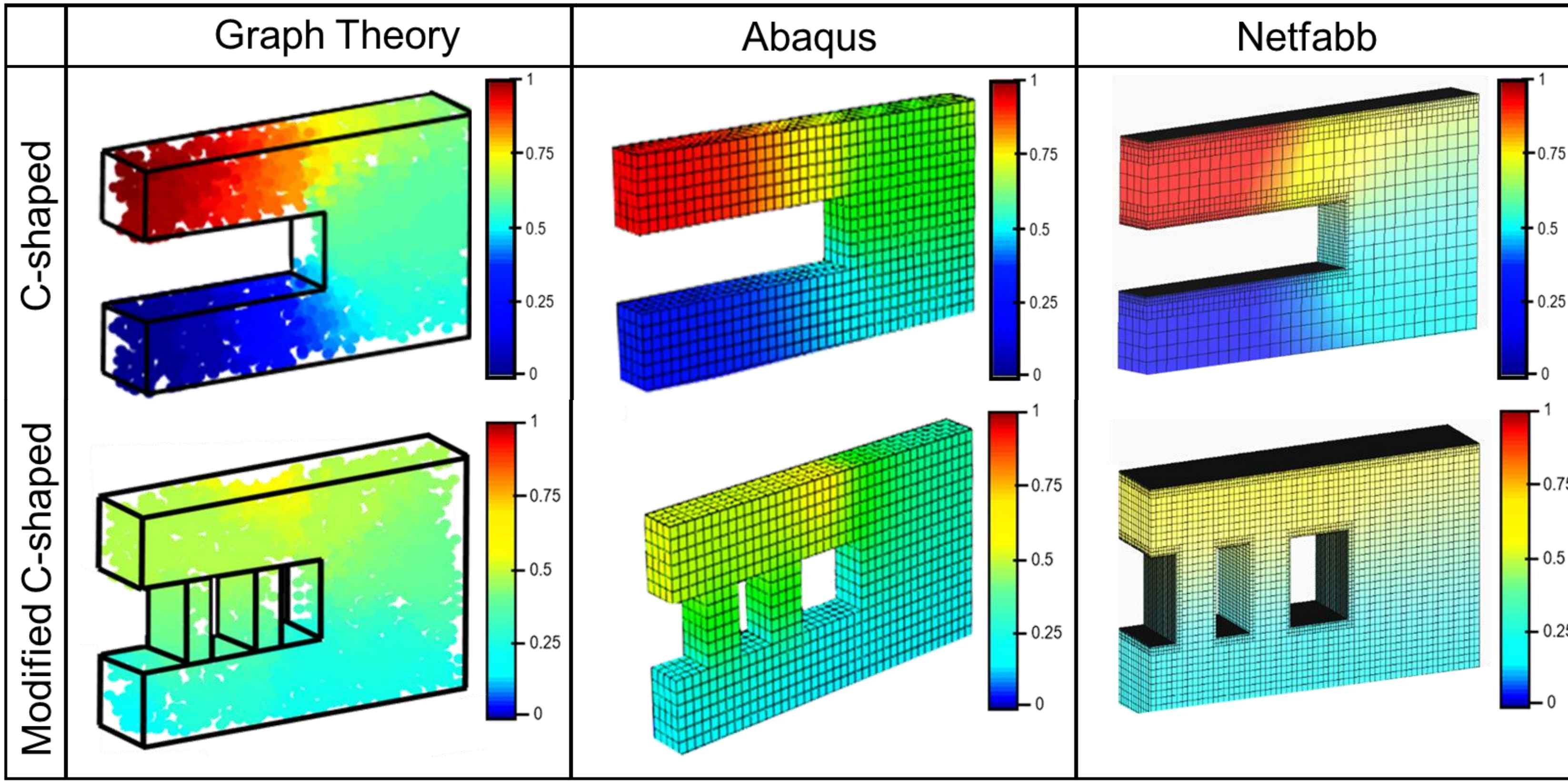






Graph theory solution converges to similar trends as finite element analysis.

Heat Distribution Comparison with Commercial Software



Error (SMAPE)	Total number of nodes	Graph theory approach time	FE analysis time
16%	1,000	0.5 min	200 min (2,000 elemnts)
10%	5,000	18 min	
8%	8,000	41 min	

Outline

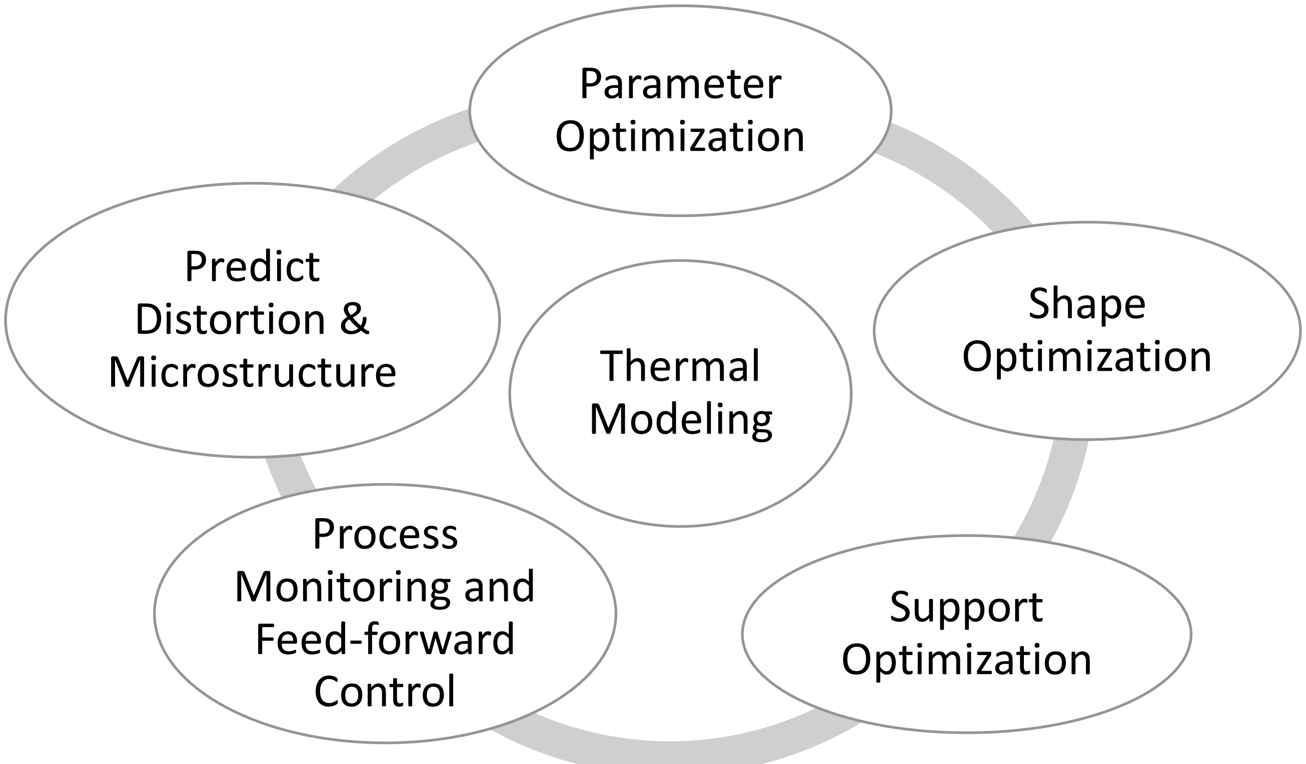
- Introduction
  - Engineering problem
  - AM thermal simulation categories
  - Solving heat equation using graph theory
- Graph theory approach in AM
- Result
  - Simulation vs exact analytical
  - Simulation vs finite element
- Conclusions and Future Work

Conclusion

- Graph theory simulates the thermal heat field within 1/10th of the time and error less than 10% of FEA.
- Validation the graph theoretic approach with experimental data (Tomorrow presentation 1400 – 1420 at Salon A)

Future Works

- Use graph theoretic thermal filed to predict part distortion.
- Use graph theoretic thermal filed to predict microstructure.



University of Nebraska Lincoln

NEBRASKA ENGINEERING
ADDITIVE TECHNOLOGY LABS

LAMPS Laboratory for Advanced Manufacturing Processes and Sensing

Speaker Information

Reza Yavari
Mechanical and Materials Engineering

University of Nebraska-Lincoln
mrezayavari89@huskers.unl.edu

https://engineering.unl.edu/lamps/